How to use long multiplication *(multiplying by a two-digit number)*

- For multiplication by 10, 11 and 12 we can use our times table knowledge (or a multiplication square for times tables we do not know) to work out the answer.
- **LONG MULTIPLICATION** is used when we want to multiply a two-digit number (or bigger) by another number bigger than 12.

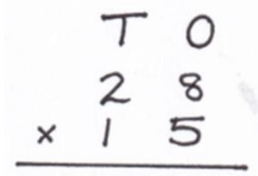
Example 1: Multiplying a 2-digit number by another two-digit number
(with exchanging and regrouping)

- Read the number sentence:

28x15=

Twenty-eight multiplied by fifteen equals

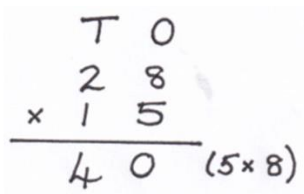
- Rewrite the number sentence in place value columns. It is important to write the bigger number first and place the smaller two-digit number underneath, with only the top line of the equals sign drawn.



METHOD 1 Long Multiplication showing partial solutions

(This method is most useful when first doing long multiplication because it helps us to learn the order in which long multiplication should be worked out and we do not need to exchange and regroup numbers until the end stage).

- We must multiply every digit in the top number (in this case **8 and 20**) by every digit in the bottom number (in this case **5 and 10**). Start with the ***ones*** column: **5x8=40** and record **5x8** in brackets to the right-hand side of the calculation, close to where the answer will be written. Write the answer **40** on the top row of the calculation.



- Staying with the ***ones*** digit in the bottom number (in this case **5**) we must now multiply it by the ***tens*** digit in the top number. Because **2** is written in the ***tens*** column its value is **20**. So **5x20=100**. Record **5x20** in brackets to the right-hand side of the calculation, close to where the answer will be written, and write the answer **100** on the second row of the calculation.

		T	O	
		2	8	
×		1	5	
		4	0	(5×8)
	1	0	0	(5×20)

- Now that we have multiplied all digits in the top number by the ***ones*** digit in the bottom number, we must now move on to the ***tens*** digit in the bottom number. The ***tens*** digit in the bottom number (in this case **10**) must now be multiplied by the ***ones*** digit in the top number (in this case **8**). **10x8=80**, so record **10x8** in brackets to the right hand side of the calculation, close to where the answer will be written, and the answer **80** on the third row of the calculation.

		T	O	
		2	8	
×		1	5	
		4	0	(5×8)
	1	0	0	(5×20)
		8	0	(10×8)

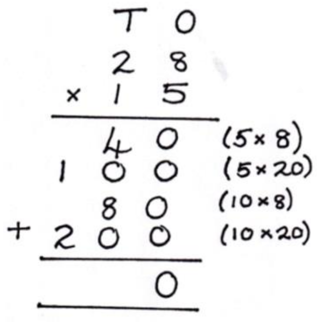
- The ***tens*** digit in the bottom number (in this case **10**) must now be multiplied by the ***tens*** digit in the top number (in this case **20**). **10x20=200**, so record **10x20** in brackets to the right hand side of the calculation, close to where the answer will be written, and the answer **200** on the fourth row of the calculation.

		T	O	
		2	8	
×		1	5	
		4	0	(5×8)
	1	0	0	(5×20)
		8	0	(10×8)
	2	0	0	(10×20)

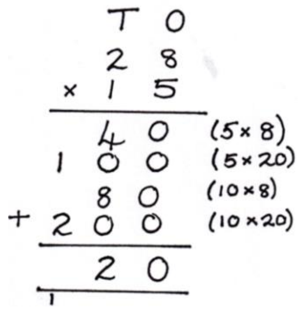
- Each row of this calculation is a ***partial solution*** to the whole ***long multiplication***. The answer can be worked out by adding the answers together. It is here that we may need to ***exchange*** and ***regroup*** numbers. First place the add sign to the left of the calculation and draw the equals sign, where the answer will be written.

		T	O	
		2	8	
×		1	5	
		4	0	(5×8)
	1	0	0	(5×20)
		8	0	(10×8)
+	2	0	0	(10×20)

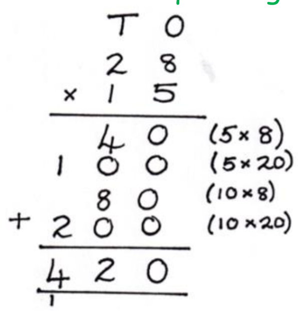
- Starting in the ***ones*** column, add the four ***partial solutions***. They total **0**, so add **0** to the ***ones*** column within the ***equals sign.***



- Move on to the ***tens*** column and add together the four answers. They total **12 *tens***, with a value of **120.**
  How many ***tens*** should be written in the ***tens*** column?
  The answer is **2 *tens*** (1**2**0), so add **2** to the tens column, within the equals sign.
  How many **10 *tens*** can be ***exchanged*** and ***regrouped*** as **1 *hundred***? The answer is **10 *tens*** (**1**20), so ***exchange*** **10 *tens*** and ***regroup*** them as **1 *hundred*** in the ***hundreds*** column.



- Now add together the numbers in the ***hundreds*** column, remembering the ***hundreds*** we ***regrouped*** from the ***tens*** column. The answer is **4 *hundreds*** so write **4** in the ***hundreds*** column, within the equals sign.



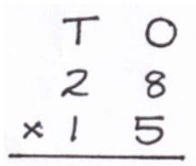
- We now have our answer within the equals sign:

28 x 15 = 420

<u>**METHOD 2**</u> <u>Long Multiplication, with exchanging and regrouping</u>

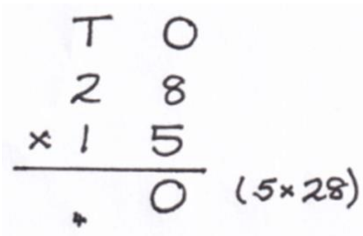
*(This method is most useful when multiplying larger numbers: up to 5-digit numbers x 2-digit numbers. You must be confident in **short multiplication**, with **exchanging** and **regrouping**, to use it).*

- The calculation is written in exactly the same way as for **METHOD 1.**



Step 1

- We start with the ***ones*** column of the bottom number (in this case **5**) and multiply it by the whole top number (**28**), using the same method as for ***short multiplication***. Record **5x28** in brackets, to the right-hand side of the calculation, close to where the answer will be written in the first row. Now multiply 5x8= and the answer is **40**. How many ***ones*** should be written in the ***ones*** column? The answer is **0** (4**0**), so write **0** in the ***ones*** column.
  How many **10** ***ones*** can be ***exchanged*** and ***regrouped*** as **1** ***ten***? The answer is **40** ***ones*** (**4**0). So ***exchange*** the **40** ***ones*** and ***regroup*** as **4** ***tens*** in the ***tens*** column.

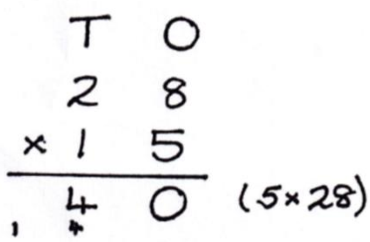


- Continuing with the ***ones*** column in the bottom number (in this case **5**), multiply it by the ***tens*** digit in the top number. Because **2** is written in the ***tens*** column its value is **20**. When we multiply, we can treat it as **2** because the answer will also be written in the ***tens*** column. So **5 x 2** ***tens*** is **10** ***tens*** (with a value of **100**). We need to remember the ***tens*** number that came from the ***ones*** column. How many ***tens*** came from the ***ones*** column? It was **4** ***tens*** so **40** must be added to **100**:

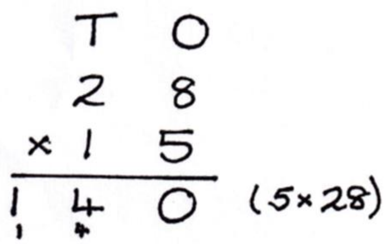
100+40=140

How many ***tens*** should be written in the ***tens*** column? The answer is 4 ***tens***. (1**4**0)

How many ***10 tens*** can be ***exchanged*** and ***regrouped*** as ***1 hundred***? The answer is ***10 tens*** (**1**40). So ***exchange*** the ***10 tens*** and ***regroup*** them as ***1 hundred*** in the ***hundreds*** column.



- As there are no other ***hundreds*** in the ***hundreds*** column, we can now put the ***regrouped*** 1 ***hundred*** into its place within the equals sign.

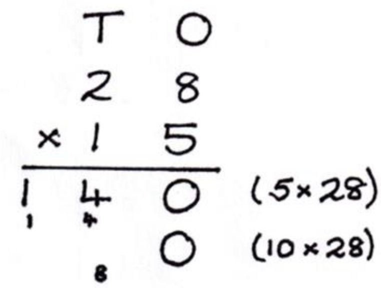


Step 2

- Now go to the ***tens*** digit in the bottom number (in this case **10**) and multiply it by the whole top number (**28**), using the same method as for ***short multiplication***. Record **10x28** in brackets, to the right-hand side of the calculation, close to where the answer will be written on the second row. Now multiply **10x8** and the answer is **80.**

  How many ***ones*** should be written in the ***ones*** column***?*** The answer is **0** (**80**), so add **0** to the ***ones*** column, in the second row.

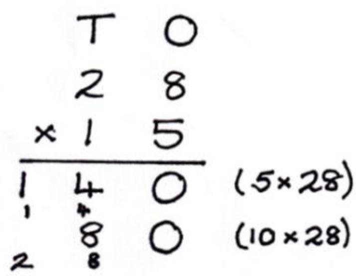
  How many ***10 ones*** can be ***exchanged*** and ***regrouped*** as ***1 ten***? The answer is ***80 ones*** (**80**). So ***exchange*** the ***80 ones*** and ***regroup*** as ***8 tens*** in the ***tens*** column.



- The ***tens*** digit in the bottom number (in this case **10**) must now be multiplied by the ***tens*** digit in the top number (in this case **20**). **10x20=200** but we need to remember that **8 *tens*** (**80**) came from the ***ones*** column, so they must be added on to **200**:

  200+80=280

  How many ***tens*** should be written in the ***tens*** column on the second row?
  The answer is **8 *tens***. (**280**)
  How many **10 *tens*** can be ***exchanged*** and ***regrouped*** as **1 *hundred***? The answer is **20 *tens*** (**2**80). So ***exchange*** the **20 *tens*** and ***regroup*** them as **2 *hundreds*** in the ***hundreds*** column.



- As there are no other ***hundreds*** in the ***hundreds*** column on the second row, we can simply put the ***regrouped* 2 *hundreds*** into the ***hundreds*** column on the second row.

$$\begin{array}{ccccl} & & T & O & \\ & & 2 & 8 & \\ & \times & 1 & 5 & \\ \hline & 1 & 4 & 0 & (5\times 28) \\ & {\scriptstyle 1} & {\scriptstyle 4} & & \\ & 2 & 8 & 0 & (10\times 28) \\ & {\scriptstyle 2} & {\scriptstyle 8} & & \end{array}$$

- We now must add together the two rows of answers, remembering to place the add sign to the left of the calculation and to draw the equals sign, where the answer will be written.

$$\begin{array}{ccccl} & & T & O & \\ & & 2 & 8 & \\ & \times & 1 & 5 & \\ \hline & 1 & 4 & 0 & (5\times 28) \\ & {\scriptstyle 1} & {\scriptstyle 4} & & \\ + & 2 & 8 & 0 & (10\times 28) \\ & {\scriptstyle 2} & {\scriptstyle 8} & & \\ \hline & & & & \\ \hline \end{array}$$

- Before adding totals, it is important to carefully cross out any numbers that were regrouped into the next column so that you do not become confused and add them a second time.

$$\begin{array}{ccccl} & & T & O & \\ & & 2 & 8 & \\ & \times & 1 & 5 & \\ \hline & 1 & 4 & 0 & (5\times 28) \\ & \cancel{\scriptstyle 1} & \cancel{\scriptstyle 4} & & \\ + & 2 & 8 & 0 & (10\times 28) \\ & \cancel{\scriptstyle 2} & \cancel{\scriptstyle 8} & & \\ \hline & & & & \\ \hline \end{array}$$

- Remember to start with adding the ***ones*** column, just as in Method 1. You will need to ***exchange*** and ***regroup*** when necessary.

$$\begin{array}{ccccl} & & T & O & \\ & & 2 & 8 & \\ & \times & 1 & 5 & \\ \hline & 1 & 4 & 0 & (5\times 28) \\ & \cancel{\scriptstyle 1} & \cancel{\scriptstyle 4} & & \\ + & 2 & 8 & 0 & (10\times 28) \\ & \cancel{\scriptstyle 2} & \cancel{\scriptstyle 8} & & \\ \hline & 4 & 2 & 0 & \\ \hline & {\scriptstyle 1} & & & \end{array}$$

- We now have our answer within the equals sign:

28 x 15 = 420

TOP TIP 1 *: When we multiply any number by* ***10,*** *the answer* ***ALWAYS*** *has a* ***0*** *in the* ***ones*** *column and all the other numbers remain in the same order but take the next higher place value (one move to the left for each digit).*

For example: ***9x10=90*** ***16x10=160*** ***28x10=280*** ***176x10=1,760***

A SPECIAL NOTE: *Example 1 has been described in detail to show how LONG MULTIPLICATION can be worked out with any numbers BUT if you already know that 28x10=280, then the answer could be written in the correct place-value positions along the correct row.*

TOP TIP *2 : When we multiply any number by a multiple of* ***10,*** *for example* ***x30***, ***x50***, ***x90*** *we can multiply the number by* ***10*** *first, by placing a* ***holding 0*** *in the* ***ones*** *column and multiplying the number by the digit in the tens column as if it was a one-digit number.*

For example: ***9x30=*** ***Step 1:*** *Put a* ***holding 0*** *in the* ***ones*** *column to multiply by* ***10***

H T O
0

Step 2: *Multiply* ***9x3=*** *and write the answer to the left of* ***0***

So 9x3=27

H T O
2 7 0

So 9x30=270

A SPECIAL NOTE: *If you understand and are confident with this method, you can use it when multiplying by multiples of* ***10*** *in LONG MULTIPLICATION*

TOP TIP *3 : When we multiply any number by a multiple of* ***100***, *for example* ***x200***, ***x500***, ***x700*** *we can multiply the number by* ***100*** *first, by placing two* ***holding 00*** *in the* ***tens*** *and* ***ones*** *columns and multiplying the number by the digit in the* ***hundreds*** *column as if it was a one-digit number.*

For example: ***4x200=*** ***Step 1:*** *Put two* ***holding 00*** *in the* ***tens and ones*** *columns to multiply by* ***100***

H T O
0 0

Step 2: *Multiply* ***4x2*** *and write the answer to the left of* ***00***

So 4x2=8

H T O
8 0 0

So 4x200=800

A SPECIAL NOTE: *If you understand and are confident with this method, you can use it when multiplying by multiples of* ***100*** *in LONG MULTIPLICATION*

Example 2: Multiplying a three-digit number by a two-digit number
(with exchanging and regrouping)

- Read the number sentence:

376x24=

Three hundred and seventy-six multiplied by twenty-four equals

- Rewrite the number sentence in place value columns. It is important to write the bigger number first and place the smaller two-digit number underneath, with only the top line of the equals sign drawn.

$$
\begin{array}{cccc}
 & H & T & O \\
 & 3 & 7 & 6 \\
\times & & 2 & 4 \\
\hline
\end{array}
$$

METHOD 1 Long Multiplication showing partial solutions

- The whole calculation is shown below:

$$
\begin{array}{ccccc}
 & & H & T & O \\
 & & 3 & 7 & 6 \\
\times & & & 2 & 4 \\
\hline
 & & & 2 & 4 & (4 \times 6) \\
 & & 2 & 8 & 0 & (4 \times 70) \\
 & 1 & 2 & 0 & 0 & (4 \times 300) \\
 & & 1 & 2 & 0 & (20 \times 6) \\
 & 1 & 4 & 0 & 0 & (20 \times 70) \\
+ & 6 & 0 & 0 & 0 & (20 \times 300) \\
\hline
 & 9 & 0 & 2 & 4 \\
\hline
 & 1 & 1 & & \\
\end{array}
$$

- We now have our answer within the equals sign:

376 x 24 = 9,024

METHOD 2 Long Multiplication, with exchanging and regrouping

*(This method is most useful when multiplying larger numbers: 3-digits, 4-digits or 5-digits x 2-digits. You must be confident in **short multiplication, with exchanging** and **regrouping**, to use it).*

- The calculation is written in the same way as for **METHOD 1** and it is shown below in two stages:

$$
\begin{array}{ccccc}
 & & 3 & 7 & 6 \\
\times & & & 2 & 4 \\
\hline
 & 1 & 5 & 0 & 4 & (4 \times 376) \\
 & 7 & 5 & 2 & 0 & (20 \times 376) \\
\end{array}
$$

- Before adding totals, it is important to carefully cross out any numbers that were regrouped into the next column so that you do not become confused and add them a second time.

$$
\begin{array}{rrrrrl}
 & & 3 & 7 & 6 & \\
\times & & & 2 & 4 & \\
\hline
 & 1 & 5 & 0 & 4 & (4\times 376) \\
+ & 7 & 5 & 2 & 0 & (20\times 376) \\
\hline
 & 9 & 0 & 2 & 4 & \\
\hline
\end{array}
$$

- We now have our answer within the equals sign:

376 x 24 = 9,024

Example 3: <u>Multiplying a 3-digit number with a decimal place by a two-digit number</u>

- When using LONG MULTIPLICATION with decimal numbers, the decimal point must be clearly written, occupying its own space:

5.73 46 =

- As we multiply the numbers, the decimal point must stay correctly aligned (*it does not move position, but remains in the same place and numbers are **exchanged** and **regrouped** around it*). Long multiplication is done just as in examples 1 and 2, remembering to ***exchange*** and ***regroup*** around the decimal point:

$$
\begin{array}{rrrrrrl}
 & & 5 & \cdot & 7 & 3 & \\
\times & & & & 4 & 6 & \\
\hline
 & 3 & 4 & \cdot & 3 & 8 & (6\times 5\cdot 73) \\
2 & 2 & 9 & \cdot & 2 & 0 & (40\times 5\cdot 73)
\end{array}
$$

- Before adding totals, it is important to carefully cross out any numbers that were regrouped into the next column so that you do not become confused and add them a second time.

$$
\begin{array}{rrrrrrl}
 & & 5 & \cdot & 7 & 3 & \\
\times & & & & 4 & 6 & \\
\hline
 & 3 & 4 & \cdot & 3 & 8 & (6\times 5\cdot 73) \\
+\,2 & 2 & 9 & \cdot & 2 & 0 & (40\times 5\cdot 73) \\
\hline
2 & 6 & 3 & \cdot & 5 & 8 & \\
\hline
\end{array}
$$

- We now have our answer within the equals sign:

5·73 x 46 = 263·58